Deterministic and Geostatistical Estimation

- Geostatistical Study Workflow
  - Traditional
  - Geostatistical
- Weighted Linear Estimators
- Kriging
  - Simple kriging
  - Kriging with External Drift
- Cross Validation
- Cokriging
- Kriging Exercises

Steps in a Traditional Geological Study

- Establish stratigraphic framework
- Establish facies, flow units and geologic prototype areas from logs and cores
- Develop log database, develop statistical model for facies, flow units and permeability from cored wells
- Map facies, flow units and geologic areas across the field with log data

Note: traditional approach does not provide a measure of uncertainty in the spatial distribution of facies flow units and associated f - k in each geologic area

Steps in a Geostatistical Study

- Establish stratigraphic framework
- Establish facies, flow units and geologic prototype areas from logs and cores
- Analyze the statistical f - k relationship in core, log and seismic data to support a facies / flow unit model in each geologic area
- Perform a spatial continuity analysis of core/log and seismic data constrained by stratigraphic framework
- Estimate (map) facies, flow units or f - k trends while honoring the stratigraphic framework, the spatial continuity, and the spatial arrangement of core/log (hard) or seismic (soft) data
- Simulate (model) facies, flow units or f - k variability while honoring the stratigraphic framework, the spatial continuity and the spatial arrangement of core/log (hard) or seismic (soft) data

Goal of Estimation

- We want our estimates to…
  - be unbiased (centered on the mean)
  - have minimum variation about the mean and
  - be based on a model

Estimation Methods

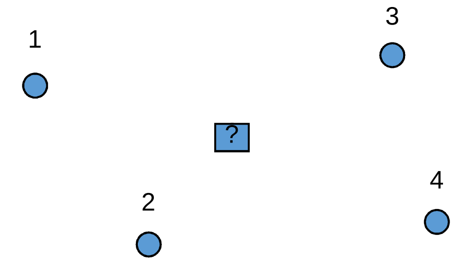
- A wide variety of estimation methods have been designed for different types of estimates:
  - Local or global estimates
  - Mean or full distribution of data values
  - Point or block values (require variograms)

Estimation Methods

- The Goal of Estimation is to Obtain the Single “Best” Value at an Unsampled Location
  - In Practice, the Estimated Value is a Function of the Algorithm (Model) Used
  - Many Algorithms Have Been Developed; Each Have Advantages and Disadvantages Compared to the Others
- All Estimation Methods Involve a Weighted Linear Combination of Sample Data Values. That is,

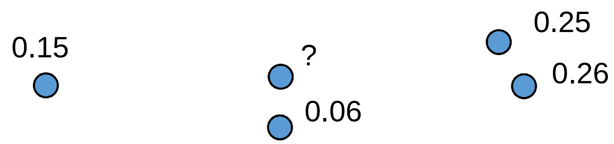
  where $z(x_i)$ = Sample Data Value at Location z_i, λ_i = Weight Assigned to $z(x_i)$, and z^* = Estimated Value at Location z_i

$$z^* = \sum_{i=1}^{n} \lambda_i z(x_i)$$



Estimation Exercise

- What is the porosity at the unsampled location?



- What do we need to consider?

Local Estimation

- Estimation Algorithms Can Be Classified In Many Ways. One Useful Classification Is
  - Global Estimation
    - Estimate Value Over Large Area (Volume)
    - Consider Data Within Area (Volume) to Be Estimated
  - Point Estimation
    - Estimate Value Over Small Area (Volume)
      - Point Values
      - Block Values
    - Consider Data Outside Area (Volume) to be Estimated

- Point Estimation Methods
  - Geological Experience and/or Artistic License
  - Traditional Algorithms That Use Weights Based on Euclidean (Geometric) Distance
    - Polygon Method (Nearest Neighbor)
    - Triangulation
    - Local Sample Mean
    - Inverse Distance
  - Geostatistical Algorithms That Use Weights Based on “Structural” / “Geological” (or Statistical) Distance
    - Simple Kriging
    - Ordinary Kriging
    - Universal Kriging
    - Kriging with Trend
    - Collocated Cokriging

- Problems Affecting All Point Estimation Methods
  - How to Weight Samples
  - Search Neighborhood
  - Data Clustering / Redundancy
- For All Point Estimation Methods:
  - Estimate (z*) is a Weighted Linear Combination

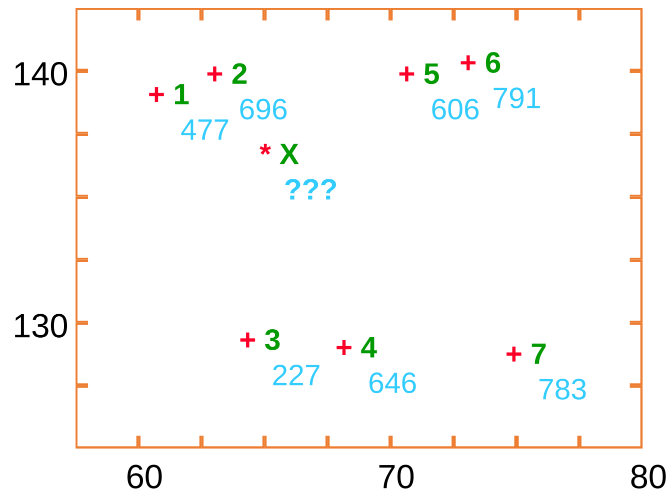
$$z^* = \sum_{i=1}^{n} \lambda_i z(x_i)$$

  - Unbiasedness Condition Generally Given by

$$\sum_{i=1}^{n} \lambda_i = 1$$

Example Data Set

Well	X	Y	Value	Distance to X
1	61	139	477	4.47
2	63	140	696	3.61
3	64	129	227	8.06
4	68	128	646	9.49
5	71	140	606	6.71
6	73	141	791	8.94
7	75	128	783	13.45



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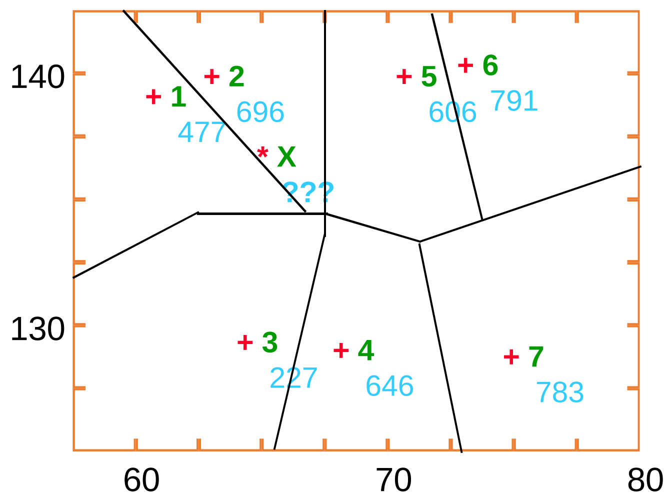
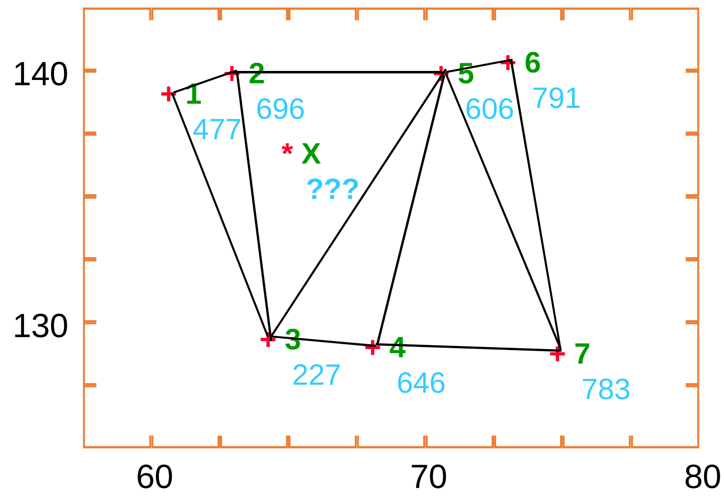
- Polygon (Nearest Neighbor) Method
  - Assign All Weight To Nearest Neighbor (Well 2 In This Case)
  - Use Perpendicular Bisectors to Divide Into Regions. Note that X Is Closest to Well 2
  - Estimated Value = 696



- Polygon (Nearest Neighbor) Method
  - Advantages
    - Easy to Use
    - Quick Calculation in 2D
  - Disadvantages
    - Discontinuous Estimates
    - Edge Effects / Sensitive to Boundaries
    - Difficult to Do in 3D

- Triangulation
  - Three Samples Receive All the Weight (Wells A, B, and E)
  - Three Weights Are Proportional to the Area of the Opposite Sub-Triangle (See Next Page for Details)
  - Estimated Value = 548.7



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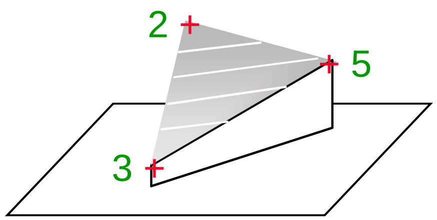
- Triangulation (continued)
  - Can Also Calculate Estimate from the Equation of the Plane that Passes Through Points 2, 5, and 3
  - Equation of Plane Derived From Solution of Three Simultaneous Equations

    $63a + 140b + c = 696$

    $64a + 129b + c = 227$

    $71a + 140b + c = 606$



  - Solution to Set of Equations

    $a = -11.250, b = 41.614, c = -4421.159$

  - Equation Used to Estimate Value of <u>Any Point</u> On Plane 2-5-3 Is

    $z^* = -11.25x + 41.614y - 4421.159$ *where x, y are location coordinates*

  - Estimate for Point X = 548.7

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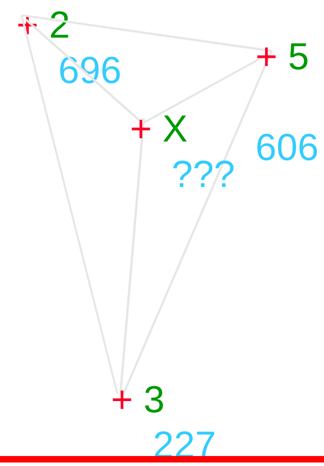
- Triangulation (continued)
  - Calculation of Weights by Area
    - Weight of 2 = Area 5X3 / Area 253 = 0.511
    - Weight of 5 = Area 2X3 / Area 253 = 0.273
    - Weight of 3 = Area 2X5 / Area 253 = 0.216
  - Estimate = (.511)(696) + (.273)(227) + (.216)(606) = 548.7



Triangle 2-5-3
Divided to Show
Sub-Triangles
Used in Calculation

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Triangulation (continued)

Advantages of Method

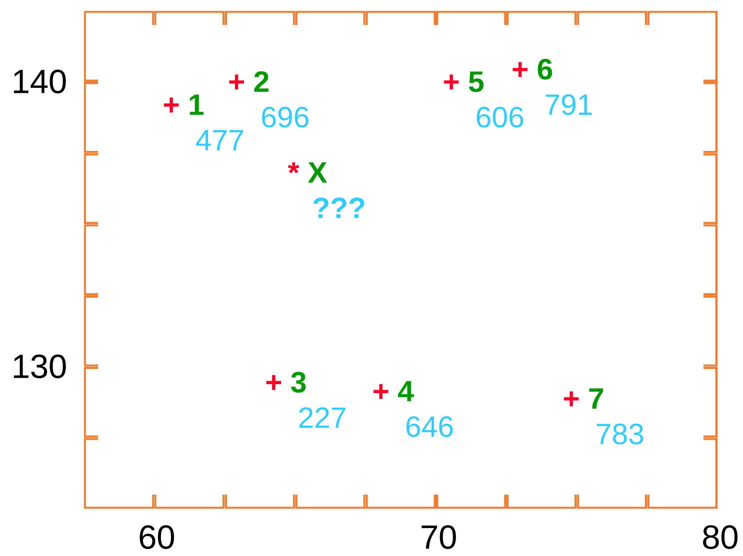
- Easy to Understand
- Fast Calculations in 2D
- Can Be Done Manually

Disadvantages

- Triangulation Network Is Not Unique. The Use of Delaunay Triangles Is an Effort to Work With A “Standard” Set of Triangles
- Not Useful for Extrapolation
- Difficult to Implement in 3D

Local Sample Mean

- All Samples Weighted Equally Within Local Neighborhood
- If All Data Shown Are In the Local Neighborhood the Estimate = 603.7



Local Sample Mean (continued)

- Advantages
  - Easy to Understand
  - Easy to Calculate in Both 2D and 3D
  - Fast
- Disadvantages
  - Local Neighborhood Definition is Not Unique
  - Location of Samples is Not Used Except to Define Local Neighborhood
  - Sensitive to Data Clustering
  - Does Not Honor the Data. At Data Locations, This Method Does Not Return the Data Value
- This Method Is Rarely Used!

- Inverse Distance Methods
  - Sample Weight is Inversely Proportional to Some Exponent of the Distance Between the Sample and the Point Being Estimated
  - The Estimate is Given By

$$z^* = \frac{\sum_{i=1}^{n} \left(\frac{1}{d^p} \right) \left(z(x_i) \right)}{\sum_{i=1}^{n} \left(\frac{1}{d^p} \right)}$$

  - *where d = distances, z(x) = sample values, p = exponent*
  - Note: Local Sample Mean is Equivalent to Exponent = 0 and the Nearest Neighbor Method is Equivalent to Exponent = Infinity.

Inverse Distance Methods (continued)

- For Example Data, the Calculation Yields Using an Exponent of 1, 2, and 3 Yields

Exp = 1

Well	X	Y	Value	Distance to X	1/d	w = (1/d)/[Sum of (1/d)]	w * Data Value
1	61	139	477	4.47	0.2236	0.2098	100.0938
2	63	140	696	3.61	0.2774	0.2603	181.1513
3	64	129	227	8.06	0.1240	0.1164	26.4224
4	68	128	646	9.49	0.1054	0.0989	63.9021
5	71	140	606	6.71	0.1491	0.1399	84.7755
6	73	141	791	8.94	0.1118	0.1049	82.9918
7	75	128	783	13.45	0.0743	0.0698	54.6168
					1.0656	1.0000	593.9537

Exp = 2

Well	X	Y	Value	Distance to X	1/d	w = (1/d)/[Sum of (1/d)]	w * Data Value
1	61	139	477	4.47	0.0500	0.2582	123.1502
2	63	140	696	3.61	0.0769	0.3972	276.4476
3	64	129	227	8.06	0.0154	0.0794	18.0326
4	68	128	646	9.49	0.0111	0.0574	37.0627
5	71	140	606	6.71	0.0222	0.1147	69.5356
6	73	141	791	8.94	0.0125	0.0645	51.0544
7	75	128	783	13.45	0.0055	0.0285	22.3373
					0.1937	1.0000	597.6204

Exp = 3

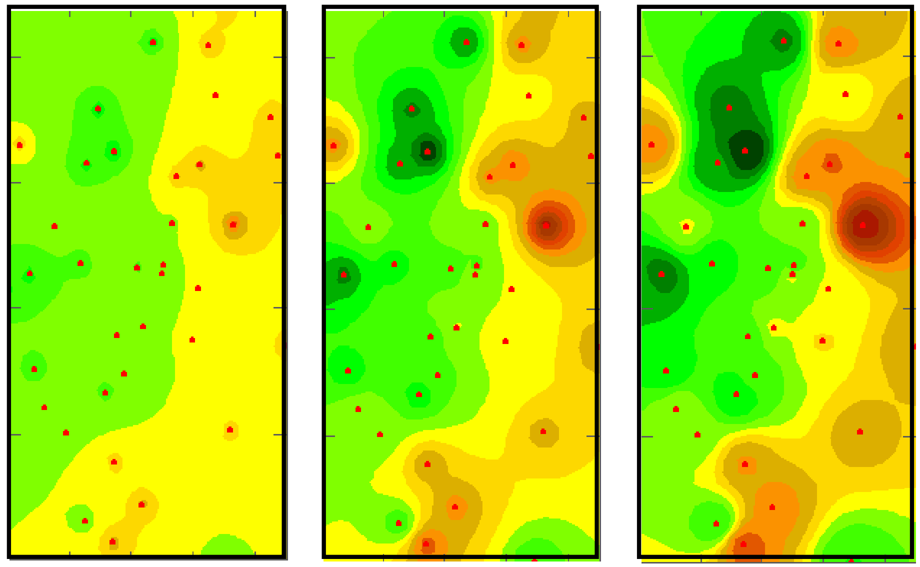
Well	X	Y	Value	Distance to X	1/d	w = (1/d)/[Sum of (1/d)]	w * Data Value
1	61	139	477	4.47	0.0112	0.2746	130.9832
2	63	140	696	3.61	0.0213	0.5240	364.7007
3	64	129	227	8.06	0.0019	0.0469	10.6389
4	68	128	646	9.49	0.0012	0.0288	18.5828
5	71	140	606	6.71	0.0033	0.0814	49.3056
6	73	141	791	8.94	0.0014	0.0343	27.1509
7	75	128	783	13.45	0.0004	0.0101	7.8974
					0.0407	1.0000	609.2596

Inverse Distance Methods (continued)

- Advantages
  - Easy to Understand
  - Easy to Implement
  - Changing Exponent Adds Some Flexibility to Adapt method to Different Estimation Problems
  - This Method Can Handle Anisotropy
- Disadvantages
  - Difficulties Encountered When Point to Estimate Coincides With Data Point (d = 0, Weight is Undefined)
    - Possible Solutions
      - Assign Data Value to Point to Be Estimated
      - Add Small Constant to Weights (Data Are No Longer Honored!)
  - Susceptible to Clustering

Inverse Distance Methods (continued)

- Comparison of Results Obtained From Different Exponents



Exponent = 1 Exponent = 2 Exponent = 3

Local Estimation

- Limitations of Traditional Estimation Methods
  - Weights Are Based On Arbitrary Schemes
  - No Model of Spatial Continuity Is Used
  - Estimates Are Biased Towards Clustered Data / No Accounting for Clustering!!
  - No Measure of Estimate Uncertainty
  - Estimated Field of Values Is Much Smoother Than the Underlying Random Field (Function) That Was Sampled. (This Is True For All Estimation Techniques, Including Kriging.)

ESTIMATION:

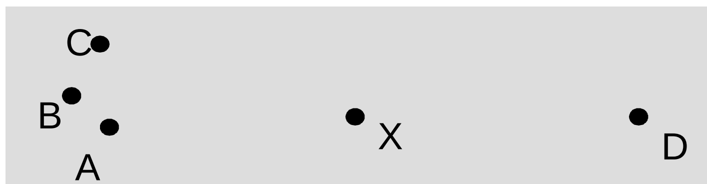
TOO SMOOTH!!! DISTRIBUTION IS

WRONG!!! SPATIAL CONTINUITY IS

WRONG!!!

Clustering

- The Effect of Clustering
  - For Methods Based on Euclidean Distances, The Estimates Will Be Biased Towards Clustered Data
  - Consider the Following Data Arrangement



  - The Estimate at X = $l_A Z(A) + l_B Z(B) + l_C Z(C) + l_D Z(D)$
  - For Inverse Distance Method All Weights Are About 0.25
  - For Kriging Method the Approximate Weights Are Given By $l_A = l_B = l_C = 0.167$ and $l_D = 0.5$

Estimation Methods

- **Kriging**
  - a minimum variance estimator based on a knowledge of variograms
  - an unbiased estimate that accounts for the data
  - provides for data declustering

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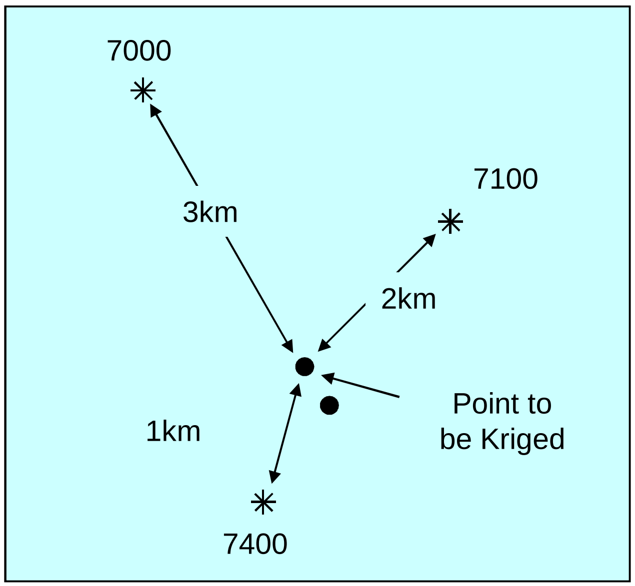
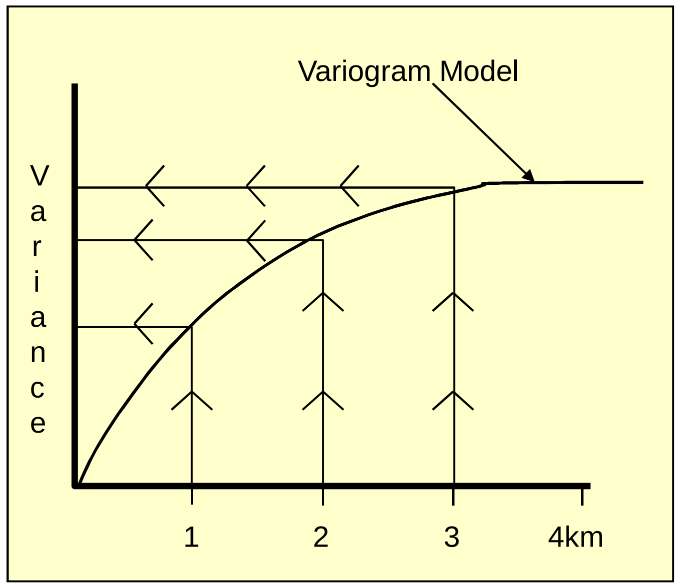
Assigning Weights

- Polygon-type estimates
- Inverse distance estimates
- Local sample mean estimates
- Local sample median estimates
- **Use variogram => kriging**

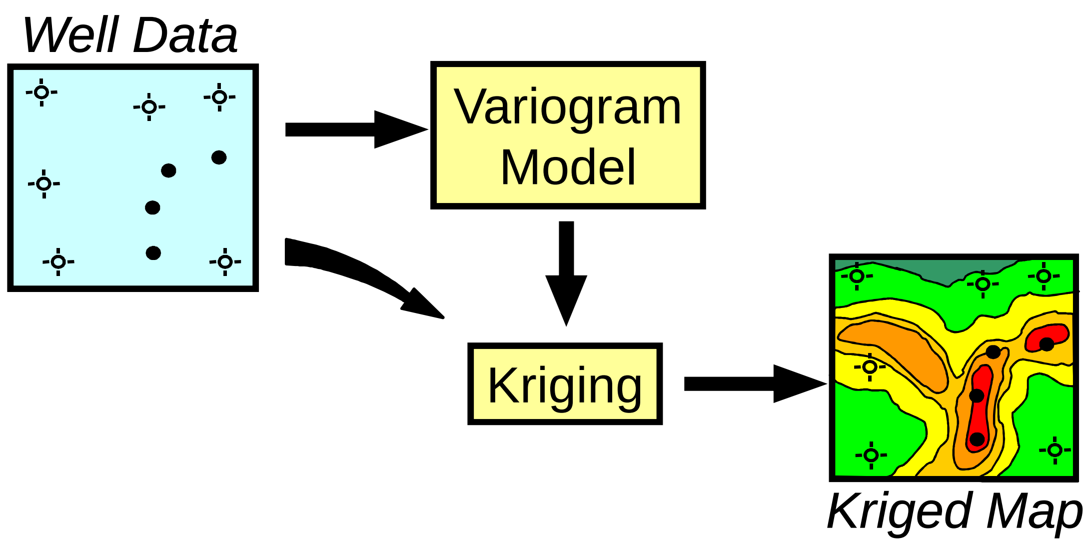
Variograms
Modeling Spatial Correlation

- Data points further away from a point to be kriged are less correlated than those closer to the point





Kriging Example using Well Data



Properties of Kriging

- Kriging provides the **<u>B</u>est <u>L</u>inear <u>U</u>nbiased <u>E</u>stimate (BLUE)**
- Kriging is an exact interpolator (kriged estimates match data value at data locations)
- Kriging system depends only on the covariance's and data configuration, not the data values
- By accounting for configuration, Kriging declusters the data
- The kriging error is uncorrelated with the kriged distribution (important for conditional simulation)
- Problems in application of kriging to reservoir modeling
  - Underrepresents the variability
  - Deterministic and cannot be used for estimation of uncertainty
  - The fields generated tend to be Gaussian

Kriging

- Kriging is a procedure for constructing a minimum error variance linear estimate at a location where the true value is unknown
- The main controls on the kriging weights are:
  - closeness of the data to the location being estimated
  - redundancy between the data
  - the variogram
- Simple Kriging (SK) does not constrain the weights and works with the residual from the mean
- Ordinary Kriging (OK) constrains the sum of the weights to be 1.0, therefore, the mean does not need to be known
- Two implicit assumptions are stationarity (work around with different types of kriging) and ergodicity (more slippery)
- Kriging is not used directly for mapping the spatial distribution of an attribute.

Types of Kriging

- Simple Kriging (SK):

$$m_{SK}^{*} = \sum_{i=1}^{n} \lambda_i \cdot z(u_i) + \left[1 - \sum_{i=1}^{n} \lambda_i\right] \cdot m_{global}$$

- Ordinary Kriging (OK)

$$m_{OK}^{*} = \sum_{i=1}^{n} \lambda_i \cdot z(u_i)$$

  - Other Types:
    - Universal Kriging (UK)
      - accounts for simple trends
    - External Drift
      - accounts for more complex trends
    - Locally Varying Mean
      - accounts for secondary information

Some Definitions

- Consider the residual data values:

$$Y\left(u_i\right)=Z\left(u_i\right)-m\left(u_i\right),\ i=1,\ldots,n$$

where m(u) could be constant, locally varying, or considered constant but unknown.

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Some Definitions

- Variogram is defined as:

$$2g\left(h\right)=E\left\{\left[Y\left(u\right)-Y\left(u+h\right)\right]^{2}\right\}$$

- Covariance is defined as:

$$C\left(h\right)=E\left\{Y\left(u\right)\cdot Y\left(u+h\right)\right\}$$

Some Definitions

- Link between the Variogram and Covariance:

$$
\begin{aligned}
2\gamma(h) &= \left[E\{Y^2(u)\}\right] + \left[E\{Y^2(u+h)\}\right] \\
&\quad - 2\cdot\left[E\{Y(u)\cdot Y(u+h)\}\right] \\
&= Var\{Y(u)\} + Var\{Y(u+h)\} - 2\cdot C(h) \\
&= 2\left[C(0) - C(h)\right]
\end{aligned}
$$

- So, $C(h) = C(0) - \gamma(h)$

Simple Kriging

- Consider a linear estimator:

$$Y^*(u) = \sum_{i=1}^{n} \lambda_i \cdot Y(u_i)$$

where $Y(u_i)$ are the residual data (data values minus the mean) and $Y^*(u)$ is the estimate

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Simple Kriging

- The error variance is defined as

$$E\left\{\left[Y^{*}(u)-Y(u)\right]^{2}\right\}=$$

$$=E\left\{\left[Y^{*}(u)\right]^{2}\right\}-2\cdot E\left\{Y^{*}(u)\cdot Y(u)\right\}+E\left\{\left[Y(u)\right]^{2}\right\}$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j E\left\{Y(u_i)\cdot Y(u_j)\right\}-2\cdot\sum_{i=1}^{n}\lambda_i E\left\{Y(u)\cdot Y(u_i)\right\}+C(0)$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j C(u_i,u_j)-2\cdot\sum_{i=1}^{n}\lambda_i C(u,u_i)+C(0)$$

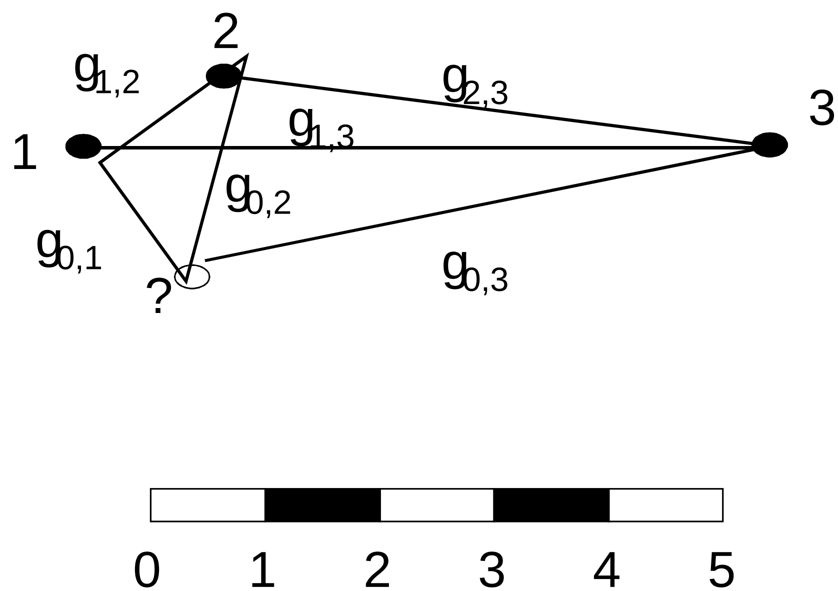
Simple Kriging System

- Optimal weights λ_i, i = 1,…,n may be determined by setting partial derivatives of the error variance w.r.t. the weights to zero

$$\frac{\partial[\ ]}{\partial\lambda_i} = 2\cdot\sum_{j=1}^{n}\lambda_j C(u_i,u_j) - 2\cdot C(u,u_i),\ i = 1,\ldots,n$$

$$\sum_{j=1}^{n}\lambda_j C(u_i,u_j) = C(u,u_i),\ i = 1,\ldots,n$$

Simple Kriging: Some Details



Simple Kriging: Some Details

- There are three equations to determine the three weights:

$$\lambda_1 \cdot C(1,1) + \lambda_2 \cdot C(1,2) + \lambda_3 \cdot C(1,3) = C(0,1)$$
$$\lambda_1 \cdot C(2,1) + \lambda_2 \cdot C(2,2) + \lambda_3 \cdot C(2,3) = C(0,2)$$
$$\lambda_1 \cdot C(3,1) + \lambda_2 \cdot C(3,2) + \lambda_3 \cdot C(3,3) = C(0,3)$$

Simple Kriging: Some Details

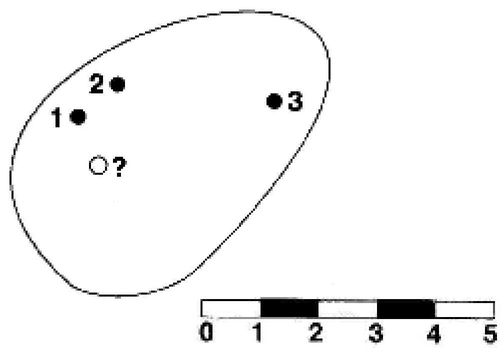
- In matrix notation:

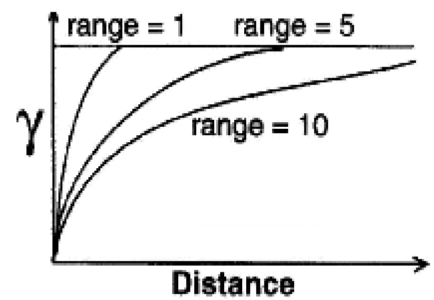
  (Recall that $C(h) = C(0) - \gamma(h)$)

$$\begin{bmatrix} C(1,1) & C(1,2) & C(1,3) \\ C(2,1) & C(2,2) & C(2,3) \\ C(3,1) & C(3,2) & C(3,3) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} C(0,1) \\ C(0,2) \\ C(0,3) \end{bmatrix}$$

Simple Kriging

Changing the Range



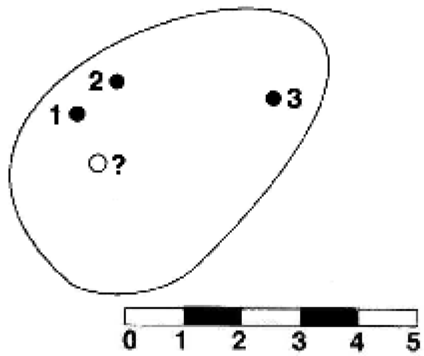


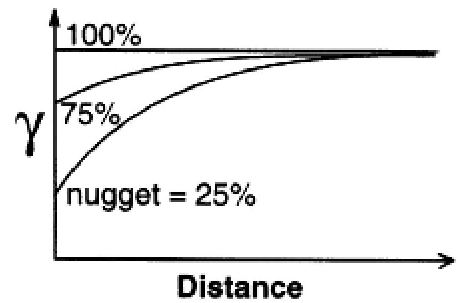
- Simple kriging with a zero nugget effect and an isotropic spherical variogram with three different ranges:

	λ_1	λ_2	λ_3
range = 10	0.781	0.012	0.065
5	0.648	-0.027	0.001
1	0.000	0.000	0.000

Simple Kriging

Changing the Nugget Effect



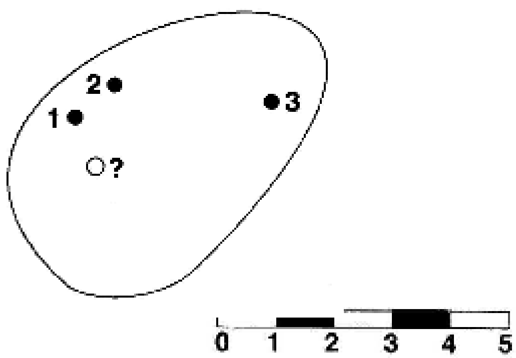


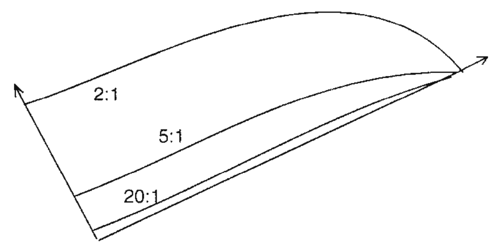
- Simple kriging with an isotropic spherical variogram with a range of 10 distance units and three different nugget effects:

	l_1	l_2	l_3
nugget = 0%	0.781	0.012	0.065
25%	0.468	0.203	0.064
75%	0.172	0.130	0.053
100%	0.000	0.000	0.000

Simple Kriging

Changing the Anisotropy





- Simple kriging with a spherical variogram with a nugget of 25%, a principal range of 10 distance units and different “minor” ranges:

	l_1	l_2	l_3
anisotropy 1:1	0.468	0.203	0.064
2:1	0.395	0.087	0.141
5:1	0.152	-0.055	0.232
20:1	0.000	0.000	0.239

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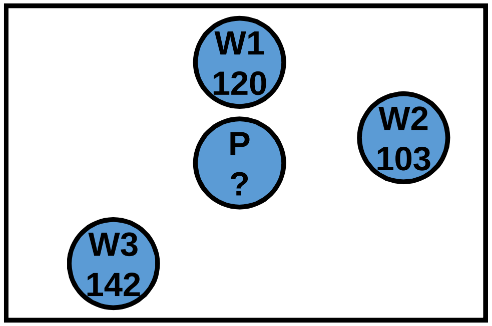
Kriging Example



Distance between Wells

	Well 1	**Well 2**	**Well 3**	**P**
Well 1	0	3.35	2.88	1.00
Well 2	3.35	0	4.79	3.32
Well 3	2.88	1.3	0	1.97
P	1	3.32	1.97	0

X/Y Position of Wells

	X	**Y**	**Value**
Well 1	3.0	4.0	120
Well 2	6.3	3.4	103
Well 3	2.0	1.3	142
P	3.0	3.0	

SemiVariance for Wells and Location P

	Well 1	**Well 2**	**Well 3**	**P**
Well 1	0	13.42	11.52	4.00
Well 2	13.42	0	19.14	13.30
Well 3	11.52	19.14	0	7.89

Kriging Example

$$\begin{bmatrix} -0.0680 & 0.0326 & 0.0354 & 0.1932 \\ 0.0326 & -0.0433 & 0.0106 & 0.4072 \\ 0.0354 & 0.0106 & -0.0461 & 0.3995 \\ 0.1932 & 0.4072 & 0.3995 & -9.5851 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ Mui \end{bmatrix} = \begin{bmatrix} 0.5954 \\ 0.0975 \\ 0.3071 \\ -0.7298 \end{bmatrix}$$

Y_p = 0.5954(120) + 0.0975(103) + 0.3071(142)
= 125.1

Error Variance = 0.5954(4) + 0.0975(12.1) + 0.3071(7.9) - 0.7298(1)
= Sqrt(5.25) = 2.3

Yp = 125.1 +/- 4.6 with 95% Probability

GEOPET

Kriging

- Kriging Varieties Can Be Distinguished According to the Model Used for the Trend m(u)
  - Simple Kriging (SK) Considers the Mean m(u) Known and Constant In the Study Area. That is,

    $m(u) = m$
  - Ordinary Kriging (OK) Accounts for Local Fluctuations of the Mean by Limiting the Domain of Stationarity of the Mean to the Local Neighborhood W(u). That is,

    $m(u') =$ Constant but Unknown
  - Kriging with Trend (KT) Considers that the Unknown Local Mean (m(u')) Smoothly Varies Within Each Local Neighborhood W(u). The Trend Component Is Modeled as a Linear Combination of Functions $f_k(u)$. That is,

    $m(u') = S\, a_k(u')\, f_k(u')$ with $a_k(u') = a_k$ Constant but Unknown

Let's look at this first!

GEOPET

Ordinary Kriging

- Ordinary Kriging - Mathematical Approach
  - An Overview of the Derivation of the Kriging Equations Is Provided On Pages 278-290 In *An Introduction to Applied Geostatistics.*
  - The Unknown Local Mean m(u) Is Filtered from the Estimator by Forcing the Weights to sum to 1!
  - The Lagrange Parameter (m) Is Used to Convert a Constrained Minimization Problem Into An Unconstrained Minimization Problem – with out too many constraints – not enough unknowns…

GEOPET

Ordinary Kriging

- OK Equations in Terms of Covariance (C)

$$\sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) C_R(u_\alpha - u_\beta) + \mu_{OK(u)} = C_R(u_\alpha - u)$$

$$\alpha = 1 \rightarrow n$$

$$\sum_{\beta=1}^{n(u)} \lambda_{\beta}^{OK}(u) = 1$$

- Error Variance

$$\sigma_{OK}^{2}(u) = C_R(0) - \sum_{\alpha=1}^{n(u)} \lambda_{\alpha}^{OK}(u) C_R(u_\alpha - u) - \mu_{OK}(u)$$

Ordinary Kriging

C Matrix D Matrix

$$\begin{bmatrix} C_{11} & C_{12} & .. & C_{1n} & 1 \\ C_{21} & C_{22} & .. & .. & 1 \\ .. & .. & .. & .. & .. \\ C_{n1} & .. & .. & C_{nn} & 1 \\ 1 & 1 & .. & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ .. \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} C_{10} \\ C_{20} \\ .. \\ C_{n0} \\ 1 \end{bmatrix}$$

Redundancy

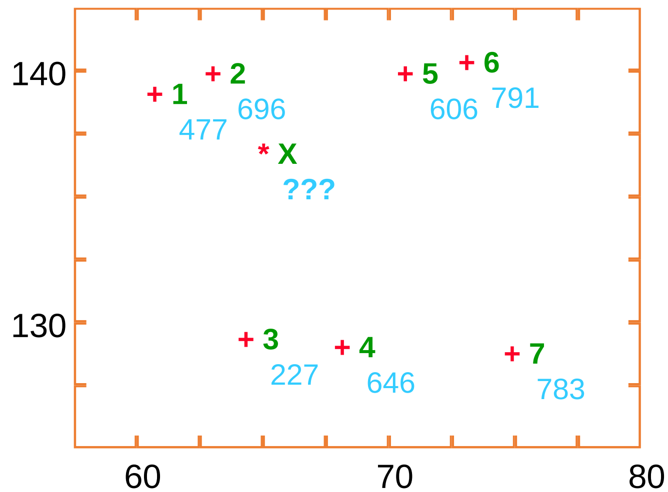
Closeness

Sum of the weights = 1

Weights

Ordinary Kriging

- Kriging - Calculation of Estimate for Example
  - Step 1 - Determine Spatial Model. For This Example Assume $C(h) = 10e^{(-0.3[h])}$. This is an Exponential Model.
  - Step 2 - Fill the C and D Matrices (Next Page)
  - Step 3 - Calculate Inverse of C Matrix and Use to Obtain Weights.
  - Step 4. - Calculate Estimate



The Kriging Matrix Setup →

Ordinary Kriging

- Kriging Example (continued)
  - Distance Matrix

Well	1	2	3	4	5	6	7
1	0.00	2.24	10.44	13.04	10.05	12.17	17.80
2	2.24	0.00	11.05	13.00	8.00	10.05	16.97
3	10.44	11.05	0.00	4.12	13.04	15.00	11.05
4	13.04	13.00	4.12	0.00	12.37	13.93	7.00
5	10.05	8.00	13.04	12.37	0.00	2.24	12.65
6	12.17	10.05	15.00	13.93	2.24	0.00	13.15
7	17.80	16.97	11.05	7.00	12.65	13.15	0.00

  - Covariance Matrix (Assumes Exponential Model)

10.00	5.11	0.44	0.20	0.49	0.26	0.05	*1.00*
5.11	10.00	0.36	0.20	0.91	0.49	0.06	*1.00*
0.44	0.36	10.00	2.90	0.20	0.11	0.36	*1.00*
0.20	0.20	2.90	10.00	0.24	0.15	1.22	*1.00*
0.49	0.91	0.20	0.24	10.00	5.11	0.22	*1.00*
0.26	0.49	0.11	0.15	5.11	10.00	0.19	*1.00*
0.05	0.06	0.36	1.22	0.22	0.19	10.00	*1.00*
1.00	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	*0.00*

GEOPET

Ordinary Kriging

- Kriging Example (continued)
  - Inverse of C Matrix (=C^{-1})

0.127	-0.077	-0.013	-0.009	-0.008	-0.009	-0.012	*0.136*
-0.077	0.129	-0.010	-0.008	-0.015	-0.008	-0.011	*0.121*
-0.013	-0.010	0.098	-0.042	-0.010	-0.010	-0.014	*0.156*
-0.009	-0.008	-0.042	0.102	-0.009	-0.009	-0.024	*0.139*
-0.008	-0.015	-0.010	-0.009	0.130	-0.077	-0.012	*0.118*
-0.009	-0.008	-0.010	-0.009	-0.077	0.126	-0.013	*0.141*
-0.012	-0.011	-0.014	-0.024	-0.012	-0.013	0.085	*0.188*
0.136	*0.121*	*0.156*	*0.139*	*0.118*	*0.141*	*0.188*	*-2.180*

  - The D Matrix (Covariances Between Data Locations and Point to be Estimated)

2.614
3.390
0.890
0.581
1.337
0.683
0.177
1

Kriging Example (continued)

- Inverse of C Matrix (C^{-1}) x D Matrix = Weights
- Note that Sum of Weights = 1.000

0.173
0.318
0.129
0.086
0.151
0.057
0.086
1.000

- Weights x Data Values = z*
- z* = Estimate = 592.3

82.491
221.184
29.223
55.812
91.583
45.273
67.163
592.729

- Kriging Example (continued)
  - Minimized Estimation Variance, s^2_R, Is Calculated Using the Equation

$$s^2_R = s^2_{DATA} - S(l_i C_{i0}) - m = 8.96$$

  - Weights x D Matrix (Covariance between Data Locations and Points Being Estimated). Note that m= -.907

Lagrange Parameter

0.050
0.202
0.039
0.015
8.956

Pick your own weights, calculate Kriging Variance – you will not do better!

Kriging

Let's look at this first!

- Kriging Varieties Can Be Distinguished According to the Model Used for the Trend m(u)
  - Simple Kriging (SK) Considers the Mean m(u) Known and Constant In the Study Area. That is,

    $m(u) = m$
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    $m(u') =$ Constant but Unknown
  - Kriging with Trend (KT) Considers that the Unknown Local Mean (m(u')) Smoothly Varies Within Each Local Neighborhood W(u). The Trend Component Is Modeled as a Linear Combination of Functions $f_k(u)$. That is,

    $m(u') = \Sigma\, a_k(u')\, f_k(u')$ with $a_k(u') = a_k$ Constant but Unknown

SK Vs OK

- Ordinary Kriging Normally Preferred Over Simple Kriging
  - Knowledge of Mean Not Required
  - Stationarity of Mean Over Entire Study Area Also Not Required
- In Areas of High Values, SK Estimates will be Lower than OK Estimates
- In Areas of Low Values, SK Estimates will be Higher than OK Estimates
- Why? Remember, OK Uses a Local Neighborhood.
  - Mean in High Value Areas will be Greater than m(u)
  - Mean in Low Value Areas will be Lower than m(u)

- Note - Both SK and OK Are Exact Estimators!

Kriging with Trend (KT)

- The Local Re-Estimation of the Mean in OK Allows One to Account for a "Global" Trend in the Data. Thus, OK Implicitly Considers a Non-Stationary RF Model Where Stationarity is Limited to the Local Neighborhood W(u').
- In Some Cases it may not be Appropriate to Even Consider the Local Mean m(u') to be Constant In Even Small Local Neighborhoods.
- Kriging with Trend (KT) Allows Modeling of the Local "Trend" Within the Neighborhood W(u) as a Smoothly Varying Function of the Coordinates.

Kriging with Trend

- Modeling the Trend
  - Trend May Be Suggested by the Physics of the Process
  - Usually, the Physics Are Not Understood Well Enough to Justify a Particular Trend. This Is the Case for Most Earth Science Data
  - Usually Choose Low Order Polynomials (Second Order or Lower)
    - Linear
      - $m(u)=m(x,y)=a + bx + cy$
    - Quadratic
      - $m(u)=m(x,y)=a+bx+cy+dx^2+ey^2+fxy$
    - Trend May Be Directional

Estimate weights and Trend Coefficients

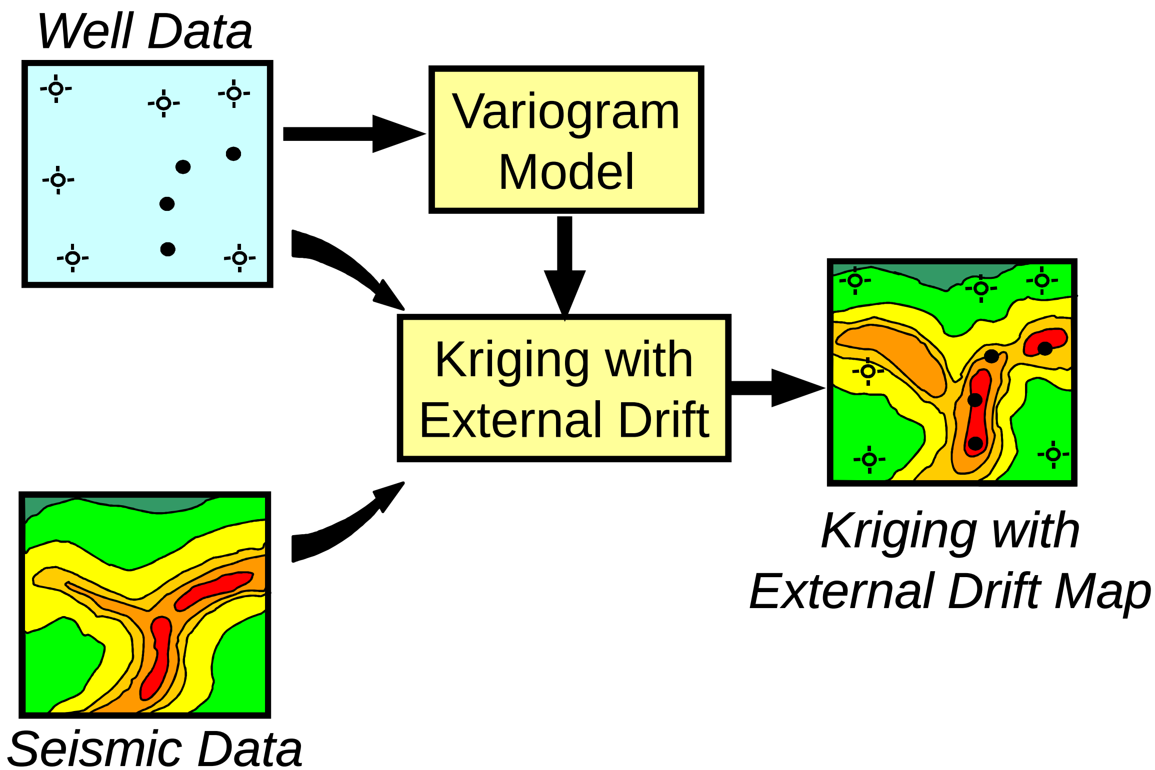
OK Vs KT

- Both OK and KT Account for a Trend In the Data. The Trend Component Is Implicit in OK and Is a Function of the Local Neighborhood. In KT the Trend Component Is Explicit.
- Any Difference Between KT and OK Is a Function of the Trend Difference
- In General, KT Will Preserve Trends Better Beyond the Limits of the Data (Extrapolation) than Will OK. OK Estimates Outside of Data Limits Approach the Local Neighborhood Mean (Example on Next Page)

Kriging with External Drift

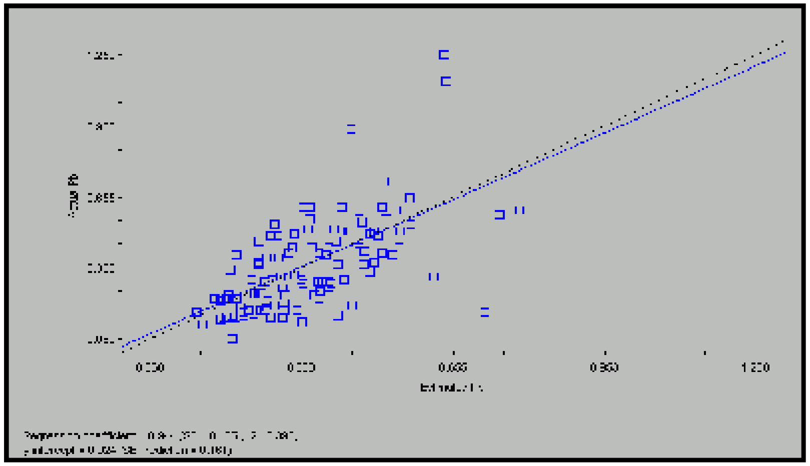
- Implicit correlation between primary and secondary (external variable)
- Requirements
  - external variable needs to vary smoothly in space (kriging system may otherwise by unstable
  - external variable must be known at all locations of the primary data and at the estimation locations
  - need to calculate and model the variogram of residuals for the primary variable
- Kriging with an external drift yields a map which reflects the spatial trend of the secondary variable

Kriging with External Drift Example using Well and Seismic Data



Cross-Validation

- Cross-validation is an attempt to check how well the estimation procedure can be expected to perform
- this is accomplished by comparing estimates with true values in production areas
- the way around this is to re-estimate each known sample value at the time for surrounding information
- cross-validation may provide the following
  - warnings, invalidation
  - reveal weaknesses, shortcomings
  - suggest improvements
  - it will never provide any guarantee



Cross-validation

- Analyze the error distribution
  - error = estimated - actual
- The items to look for are
  - averaged error (global bias)
  - spread (standard deviation)
  - maximum, minimum error
  - shape of distribution
- Plot the residual on maps
  - any persistent overestimation
  - any persistent underestimation's
- Which criterion is relevant to the study?
    - misclassification
    - minimizing underestimation
    - minimizing overestimation

Cross Validation Example

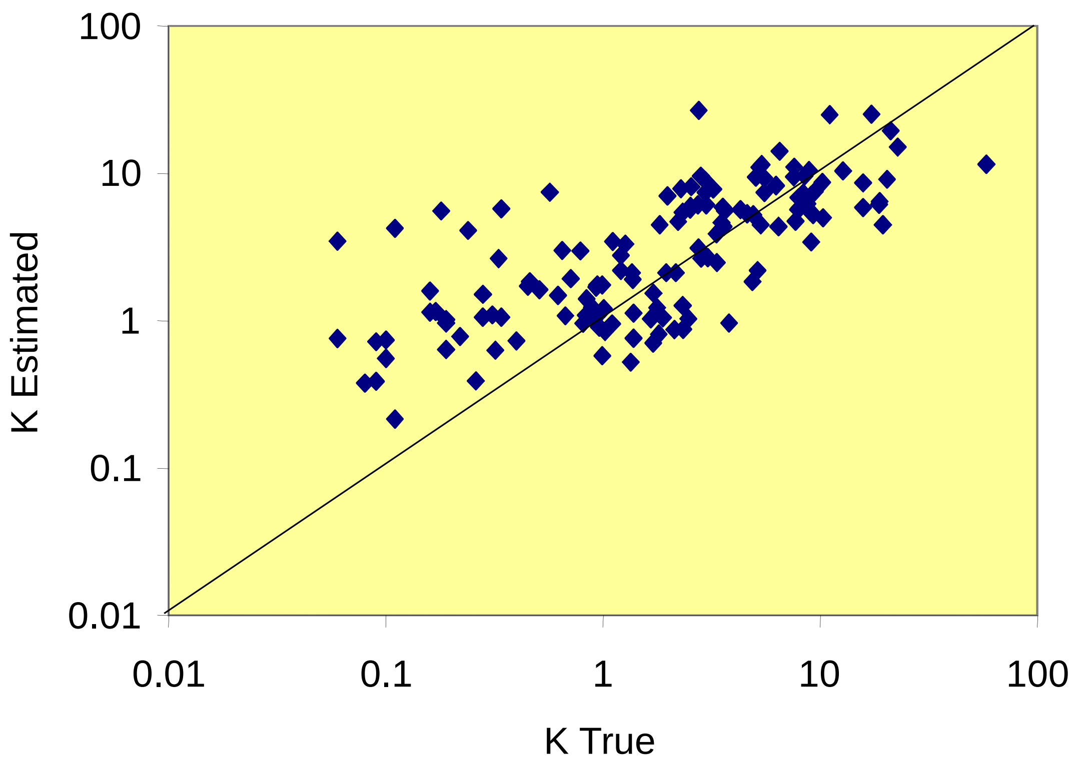
1.
Data For Five Locations

2.
Krige to Obtain Estimate at Location X

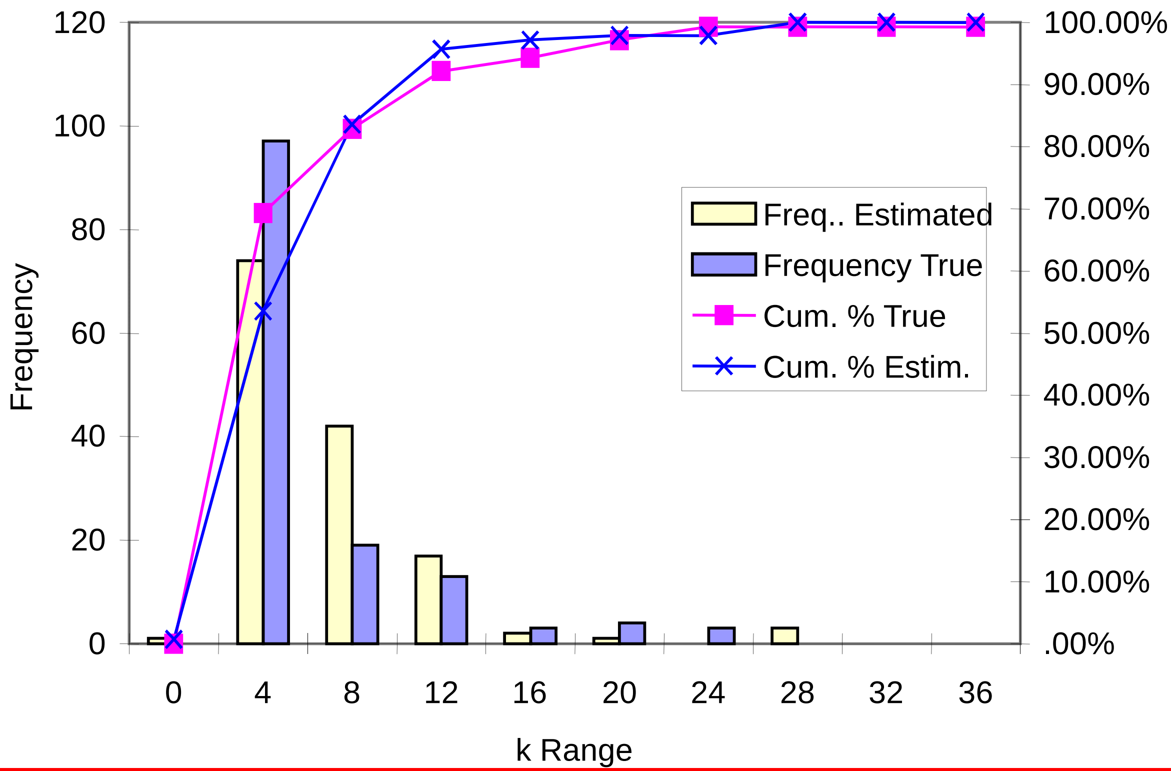
3.
Compare Estimate at X With Actual Value; Repeat Process for All Locations.

4.
Map Showing Difference Between Estimates and True (Actual) Values

Cross Validation



Cross Validation



Cross Validation

	K True	K Estimated
Mean	4.350	4.608
Standard Error	0.569	0.400
Median	2.195	3.215
Mode	0.190	1.837
Standard Deviation	6.727	4.732
Sample Variance	45.247	22.395
Kurtosis	29.837	7.203
Skewness	4.450	2.290
Range	58.260	26.958
Minimum	0.060	-0.193
Maximum	58.320	26.765
Sum	609.060	645.144
Count	140	140
Q3 - Largest (35)	5.380	6.183
Q1 - Smallest (35)	0.670	1.092
Confidence Level (95.0%)	1.124	0.791

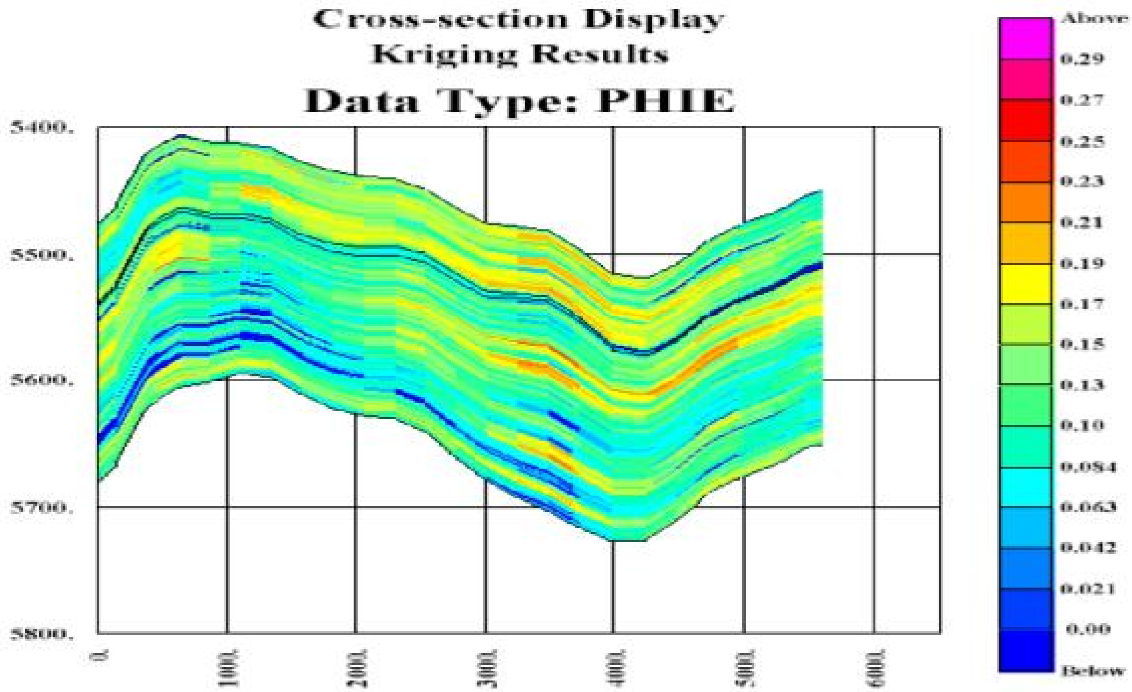
Kriging - Advantages and Disadvantages

- Advantages
  - Uses A Model of Spatial Continuity
    - Usually Derived From Sample Data
    - Analogues Are Useful In the Absence of Sufficient Real Data
  - Forces Consistency in Estimation
  - Built In Correction for Data Redundancy (Clustering)
  - Unbiased
  - Minimum Variation About the True Mean
  - Provides a Measure Error (Kriging Variance)
  - Best Linear Unbiased Estimator (or GLUE)
- Disadvantages
  - Not Easy to Understand Mathematical Details!
  - Model of Spatial Continuity (Variogram) Is Needed
  - More computationally intensive than other estimation methods
  - String Effect

Kriging - *Effective Porosity Egyptian Example*

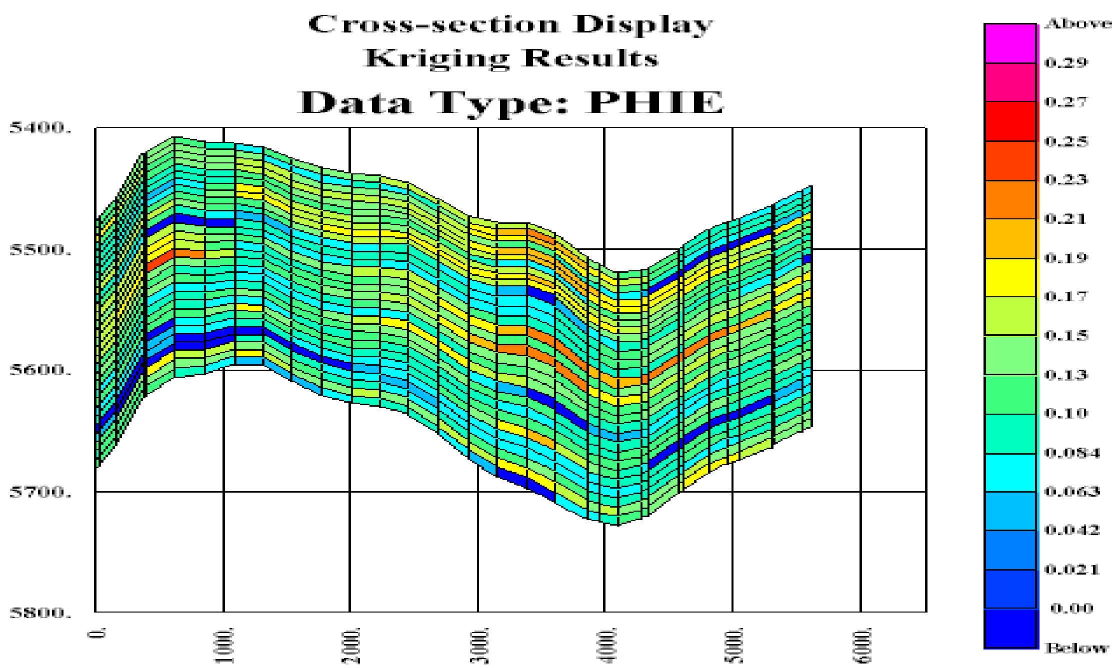
- Two porosity models:
  - High Resolution Model (geocellular model resolution)
    - 200 vertical layers
    - 320,000 Model cells
  - Low Resolution model
    - 30 vertical layers
    - 48,000 Model cells

High Resolution Model



200 vertical layers
320,000 model cells

Intermediate



32 vertical layers
48,000 Model cells

Types of Secondary Data

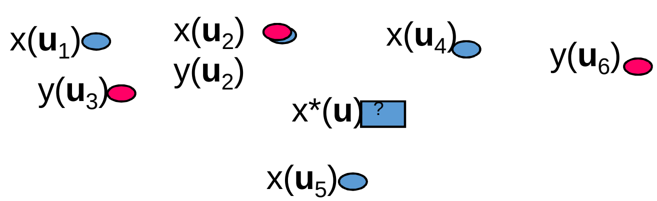
- Reservoir Property Trends
  - Linear Trend / Drift in Porosity Mean
- Categories / Groups
  - Facies with Unique Distributions and Spatial Structures
- Sparsely Sampled / Correlated Secondary Data
  - Porosity Samples Applied to Improve Permeability Estimation
- Exhaustive Secondary Data
  - Realization of Porosity Applied to Inform a Permeability Realization

Cokriging

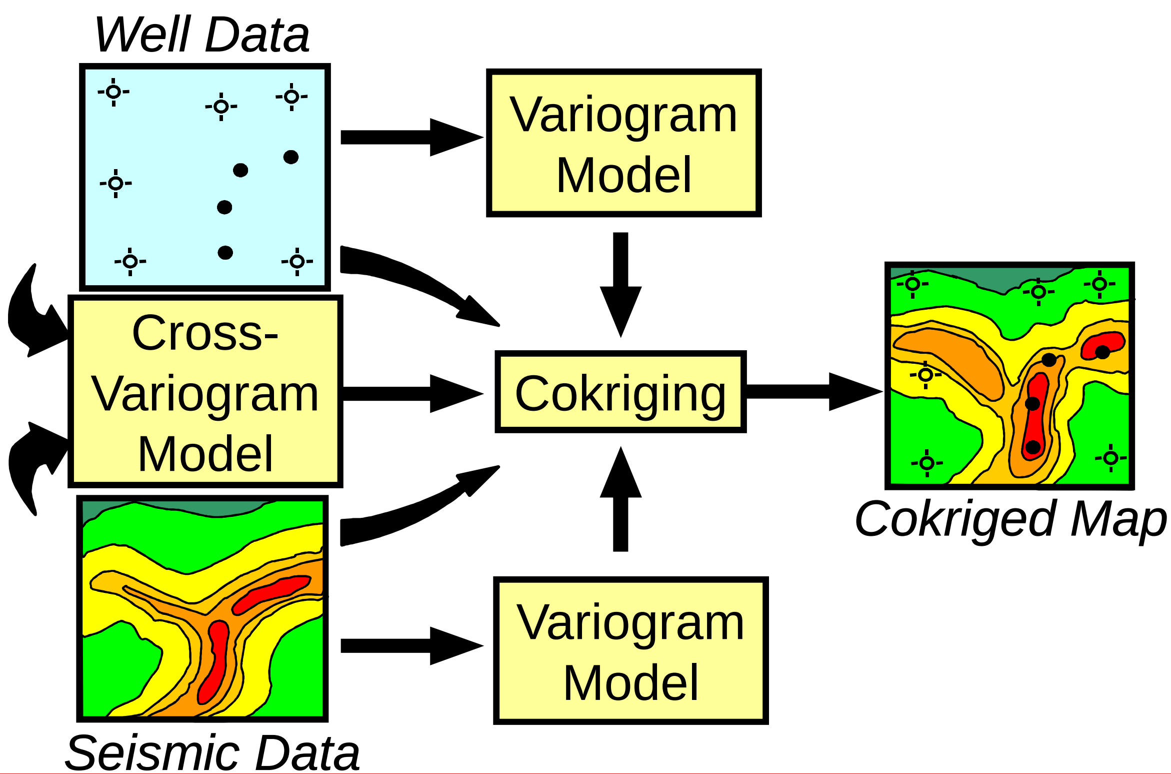
- Problem
  - Petrophysical data (e.g. porosity,permeability) is sampled sparsely (i.e. at wells)
  - Seismic data (amplitude) is sampled densely but does not directly measure desired property (e.g. porosity or permeability)
- A solution
  - Cokriging correlates desired undersampled reservoir property to widely sampled parameter

Cokriging

- Estimate primary varaible considering all relavant primary and secondary data
- Non-Exhaustive Secondary Data Can Be Incorporated Using the Cokriging Approach that Explicitly Accounts for Spatial Cross Correlation Between Primary and Secondary Variables
- Need Semivariograms (or Covariance Functions) for:
  - Primary Data
  - Secondary Data
- Need Cross Semivariograms (or Covariance Functions) Between Primary and Secondary Variables



Cokriging Example Using Well and Seismic Data



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Cokriging

- The system of equations is the same as for simple kriging with one spatial variable
- The cokriging system requires more inference of the correlation's between the different variables and their spatial correlation's
- The cokriging system requires measurement and modeling of the covariance's of each of the data types and the cross-covariance's of each data type with the others
- Cokriging is the most labor intensive option since it requires variograms of the secondary variable as well as a cross covariance.
- It is the slowest algorithm to run because the matrix is far more complicated since it must handle the additional covariance values from the primary and secondary variables as well as the cross covariance.
- Cokriging is best used when the primary variable is significantly undersampled while the secondary variable is well sampled. It is also recommended when the secondary variable is quite heterogeneous.
- Cokriging does not require, like KT or KED, that the secondary sample be smoothly varying. Neither is it required that the secondary variable exist at the primary data locations and the locations to be estimated like KED.
- The same variogram model type must be used for primary, secondary and cross variograms.

Ordinary Cokriging

- Ordinary Cokriging
  - The Ordinary Cokriging Estimator Is

$$Z_{OCK}^{1*}(u) =$$

$$\boxed{\sum_{\alpha_1=1}^{n_1(u)} \lambda_{1\alpha_1}^{OCK}(u) Z_1\left(u_{\alpha_1}\right)} + \boxed{\sum_{\alpha_2=1}^{n_2(u)} \lambda_{1\alpha_2}^{OCK}(u) Z_2\left(u_{\alpha_2}\right)} +$$

$$\boxed{\lambda_{m_1}(u) m_1} + \boxed{\lambda_{m_2}(u) m_2}$$

This is new!

  - Where the Weights Are Subject to the Constraints

$$\sum_{\alpha_1=1}^{n_1(u)} \lambda_{\alpha_1}^{OCK}(u) = 1$$

$$\sum_{\alpha_1=2}^{n_2(u)} \lambda_{\alpha_2}^{OCK}(u) = 0$$

- Or, In Terms of Covariances and Cross Covariances

$$\sum_{\beta_1=1}^{n_1(u)} \lambda_{\beta_1}^{OCK} C_{11}(u_{\alpha_1} - u_{\beta_1}) + \sum_{\beta_2=1}^{n_2(u)} \lambda_{\beta_2}^{OCK} C_{12}(u_{\alpha_1} - u_{\beta_2}) + \mu_1^{OCK}(u) =$$

$$C_{11}(u_{\alpha_1} - u), \alpha_1 = 1 \rightarrow n_1(u)$$

$$\sum_{\beta_1=1}^{n_1(u)} \lambda_{\beta_1}^{OCK} C_{21}(u_{\alpha_2} - u_{\beta_1}) + \sum_{\beta_2=1}^{n_2(u)} \lambda_{\beta_2}^{OCK} C_{22}(u_{\alpha_2} - u_{\beta_2}) + \mu_2^{OCK}(u) =$$

$$C_{21}(u_{\alpha_2} - u), \alpha_2 = 1 \rightarrow n_2(u)$$

$$\sum_{\alpha_1=1}^{n_1(u)} \lambda_{\alpha_1}^{OCK}(u) = 1$$

$$\sum_{\alpha_1=2}^{n_2(u)} \lambda_{\alpha_2}^{OCK}(u) = 0$$

- this is the system of equations we are solving when we estimate at each location.

Or, In Matrix Form

Ordinary Cokriging Constraints

Primary Covariances

Secondary Covariances

$$
\begin{bmatrix}
C^{zz}_{11} & C^{zz}_{12} & C^{zz}_{13} & C^{zz}_{14} & C^{yz}_{15} & C^{yz}_{16} & C^{yz}_{17} & C^{yz}_{18} & C^{yz}_{19} & 1 & 0 \\
C^{zz}_{21} & C^{zz}_{22} & .. & C^{zz}_{24} & C^{yz}_{25} & .. & .. & .. & C^{yz}_{29} & 1 & 0 \\
C^{zz}_{31} & .. & C^{zz}_{33} & C^{zz}_{34} & C^{yz}_{35} & .. & .. & .. & C^{yz}_{29} & 1 & 0 \\
C^{zz}_{41} & C^{zz}_{42} & C^{zz}_{43} & C^{zz}_{44} & C^{yz}_{45} & C^{yz}_{46} & C^{yz}_{47} & C^{yz}_{48} & C^{yz}_{49} & 1 & 0 \\
C^{yz}_{51} & C^{yz}_{52} & C^{yz}_{53} & C^{yz}_{54} & C^{yy}_{55} & C^{yy}_{56} & C^{yy}_{57} & C^{yy}_{58} & C^{yy}_{59} & 0 & 1 \\
C^{yz}_{61} & .. & .. & C^{yz}_{64} & C^{yy}_{65} & C^{yy}_{66} & .. & .. & C^{yy}_{69} & 0 & 1 \\
C^{yz}_{71} & .. & .. & C^{yz}_{74} & C^{yy}_{75} & .. & C^{yy}_{77} & .. & C^{yy}_{79} & 0 & 1 \\
C^{yz}_{81} & .. & .. & C^{yz}_{84} & C^{yy}_{85} & .. & .. & C^{yy}_{88} & C^{yy}_{89} & 0 & 1 \\
C^{yz}_{91} & C^{yz}_{92} & C^{yz}_{93} & C^{yz}_{94} & C^{yy}_{95} & C^{yy}_{96} & C^{yy}_{97} & C^{yy}_{98} & C^{yy}_{99} & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \mu_1 \\ \mu_2
\end{bmatrix}
=
\begin{bmatrix}
C^{zz}_{10} \\ C^{zz}_{20} \\ C^{zz}_{30} \\ C^{zz}_{40} \\ C^{yz}_{50} \\ C^{yz}_{60} \\ C^{yz}_{70} \\ C^{yz}_{80} \\ C^{yz}_{90} \\ 1 \\ 0
\end{bmatrix}
$$

Cross Covariances

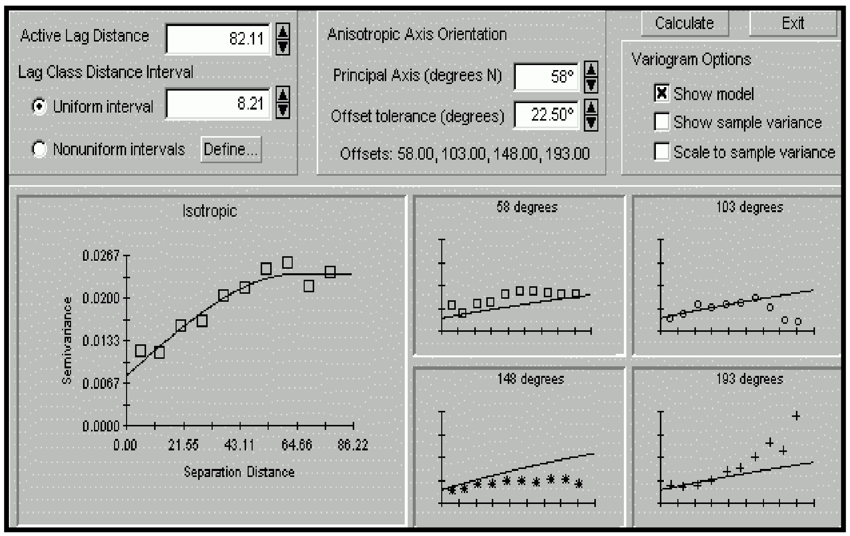
Advantages of Collocated Cokriging

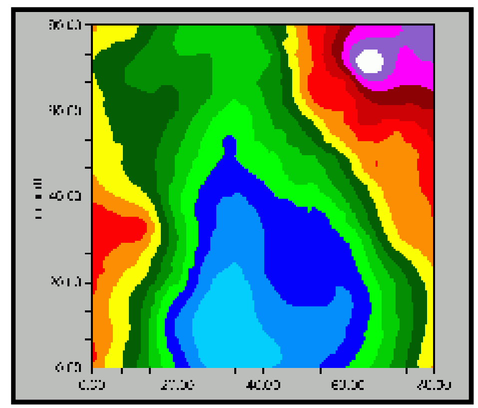
- Advantages
  - Easy to implement (as easy as external drift)
  - Includes level of correlation between hard and soft data
  - Compared to cokriging, collocated cokriging is fast because of the smaller cokriging system.
  - It doesn't require modeling the secondary attribute nor the cross variogram.
  - However, the secondary variable needs to be known at all output locations being estimated.
  - Collocated Cokriging Model for Combining Seismic and Well Log Data

Disadvantages of Collocated Cokriging

- Disadvantages
  - Collocated cokriging maps will not look like secondary variable unless there is a high correlation coefficient between the two variable
  - Secondary variable need to be sampled at all primary variable locations
  - Ignores information brought by non-collocated data beyond that of the collocated datum

Co-Kriging Exercises

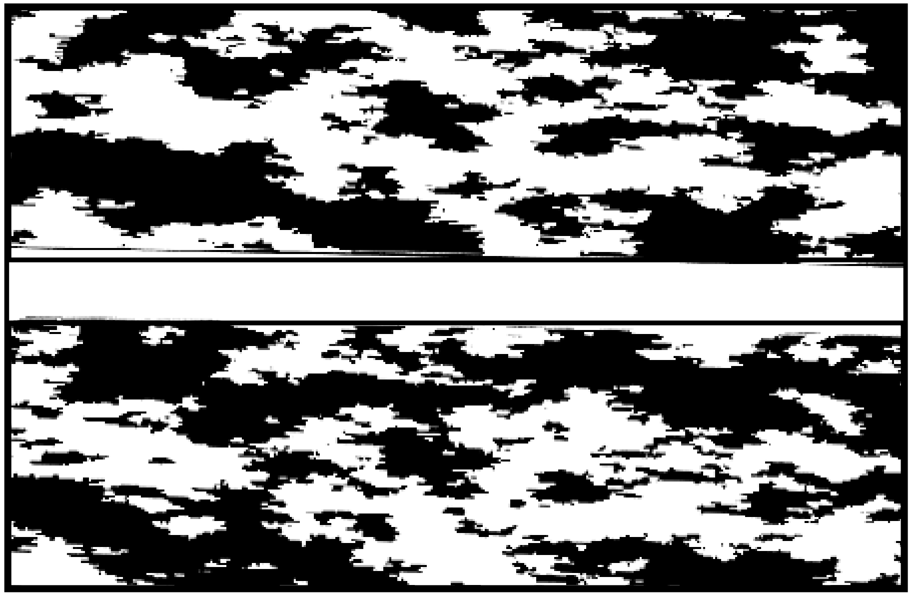




Impact of Changing the Type of Variogram Model

Spherical

Exponential

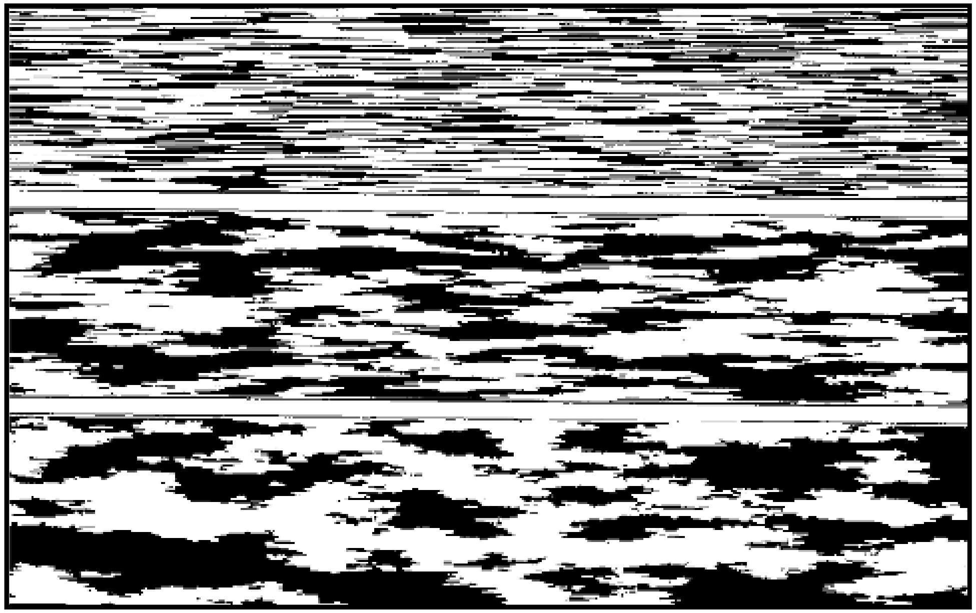


Impact of Changing the Vertical Range on the Exponential Variogram

Vertical Range 1%

Vertical Range 5%

Vertical Range 10%



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Impact of Changing the Horizontal Range on the Exponential Variogram

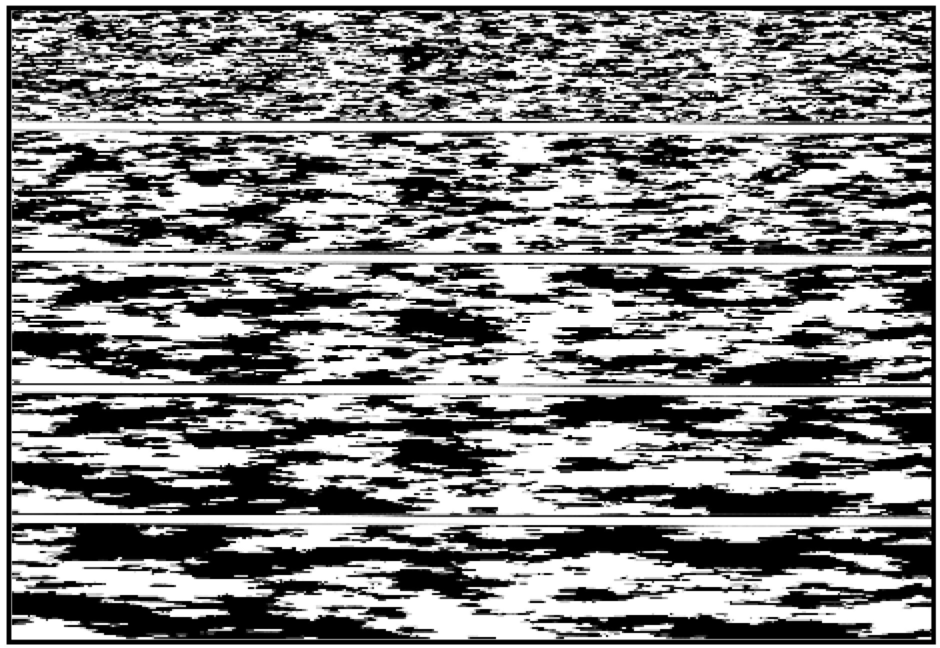
Horizontal Range 2%

Horizontal Range 4%

Horizontal Range 6%

Horizontal Range 8%

Horizontal Range 10%



The Quiz

Estimation

- What is the basic method for all estimation methods?
- What is the difference between inverse distance and kriging?
- What is the difference between simple and ordinary kriging?
- What does indicator kriging estimate?
- What is the method for cross validating a kriged map.

Exercise 3

Using the Ordinary Kriging method, estimate the value of porosity at the given location P.

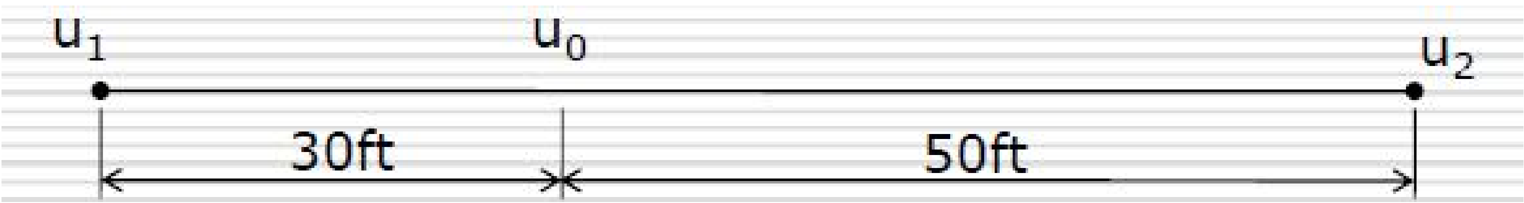
Well	X	Y	Porosity
1	871,276.03	1,157,062.00	20.149
2	868,781.64	1,156,673.00	17.979
3	869,174.71	1,156,453.00	24.375
4	868,880.48	1,156,271.00	23.332
5	869,933.78	1,156,292.00	25.073
6	870,284.11	1,156,329.00	24.637
7	870,548.96	1,156,710.00	26.256
8	870,853.51	1,156,067.00	27.099
9	870,503.30	1,156,399.00	24.503
10	869,994.58	1,155,822.00	28.227
11	869,786.45	1,155,760.00	27.589
12	870,304.50	1,155,607.00	23.984
13	870,017.09	1,155,574.00	25.321
14	870,000.00	1,156,400.00	?

Exercise 5

Consider the data configuration, x(u1)=10, x(u2)=20, and m=18. Further, assume that the isotropic variogram, representing the spatial relationship, is given by:

$$\gamma(h) = 100\left[1 - \exp\left(\frac{-3h}{100}\right)\right], h \geq 0$$

Estimate the value at location u0.



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